APPENDIX L:
REGIONAL PORTFOLIO MODEL

Contents

List of Figures and Tables

1 THE REGIONAL PORTFOLIO MODEL (RPM)

The Regional Portfolio Model (RPM) uses methods first developed for the 5th Power Plan. While the version of the model used to develop the Seventh Plan differs in some respects from this methodology, the rationale behind the core logic and analytical approaches taken in the RPM is documented in the 5th and 6th Power Plans. In particular, see appendices L and P in the 5th Power Plan and appendix J in the 6th Power Plan.

While this previous RPM documentation does not capture the exact implementation of the current model, it is an essential part of understanding the evolution of this model. This appendix does not attempt to restate the underlying rationale behind the methods used. Rather, it attempts to concisely yet comprehensively document the methods used in the version of the model employed to develop the 7th Plan. This appendix documents the functionality used for Seventh Plan. This appendix is not intended to provide a comprehensive documentation of what is possible to analyze using RPM. The platform on which the RPM model is built is extremely adaptive. Any attempt at documenting what is possible with the model would quickly be outdated. The ultimate documentation for what is possible in the model is contained within the model software itself

This appendix sets out the underlying mathematical, statistical and economic theory for this model. It is designed for readers with a strong background in system science, statistics, engineering or a similarly quantitative field. A more general description of the RPM, input assumptions, scenarios tested and results see Chapters 3 and 15. Because this appendix is intended for reference, many elements are repeated in anticipation it being read as one or several sections at a time rather than from start to finish.

2 RPM FUTURE DISTRIBUTION SIMULATION

RPM uses several statistical modeling approaches to generate a distribution of forecast time series from a set of reference forecasts. The distribution of forecasts is then used to assess the risk of different potential "futures". Thus, the RPM uses an embedded Monte Carlo simulation to test each Resources Strategy and assess the distribution of system costs across a wide range of potential future conditions. Collectively, the sub-models within the RPM that produce these forecasts are referred to as "Risk Models." The mathematical basis for the risk models are described in the following section.

2.1 Risk Models

The risk models used in the Monte Carlo simulation take the general form of some functional relationship to a simulated statistical distribution. This model form is very commonly used in many finance and engineering applications. While RPM taken as an entire methodology is a cutting edge portfolio model, its risk model components are relatively basic. Similar models and even more complex models are used in many applications to power systems including forecasting for portfolio or production cost models.

The risk models generally have two output dimensions, time and future or game. Time will be denoted by t and is a forward looking index. That is, for the 7th plan, at $t = 1$ the time is the fourth

quarter of 2015 at $t = 2$ the time is the first quarter of 2016, etc. In the 7th plan there are 80 quarters forecast, thus $t = 1\ to\ 80$ is the range for time. The future or game is a concept used in Monte Carlo simulations and will be denoted by i. In the 7th plan there are 800 futures, thus $i = 1\ to\ 800$ is the range. The futures are developed by repeatedly generating random variables, and computing a set of risk values at time t. The interpretation of set of futures at a particular time t, can be either 800 possible realizations of a particular time t, or 800 potential forecast futures at a time t.

Most of the risk models have very similar mechanics with the main differences being the parameters used. To illustrate the places where these mechanics are similar, the same mathematical notation has been adopted to describe these models. Each of these models is developed with independent random draws from simulated distributions. When reading the following sections it should be understood that every parameter and every distribution is a function of the risk model being described. It would be more explicit notation to show each parameter, term and distribution as a function of risk model being described, e.g. $\alpha_F(Load\ Risk)$, $P_{t,i}(Peak\ Demand\ Ratio)$ or $\theta_{F,i}(Natural\ Gas\ Price\ Risk)$. For the sake of brevity and readability this notation is excluded and considered implicit in the description of the risk models.

2.1.1 Load Risk

The RPM load model modifies a reference forecast that is input into the model. Let $d_{Flat}(t)$ be the forecast for flat (aMW) electric load at time (or period) t. Then the forecast for future or game i is modified by two terms. The first term is defined as follows:

$$P_{t,i} = e^{\alpha_F \theta_{F,i} + \alpha_L \theta_{L,i}(y_t - y_0) + \alpha_Q \theta_{Q,i}(y_t - y_0)^2} \quad [1,2]$$

where $y_t = year\ at\ time\ t$; α_F, α_L and α_Q are parameters; and $\theta_{F,i} \sim \theta_{L,i} \sim \theta_{Q,i} \sim N(0,1)$, that is they are independent standard normal random variables. The second term is

$$S_{t,i} = e^{\tau_{q_t} * \varepsilon_{W,i,t}} \quad [3,4]$$

where τ_{q_t} are parameters that change by quarter where $q_t = quarter\ at\ time\ t$ and $\varepsilon_{W,i,t} \sim N(0,1)$ a normal random variable.

Given these terms, the load risk for future i at time t, $D_{Flat}(t,i)$ is

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[1] The random variables in this factor do not depend on time, so they are fixed throughout the simulation. This in part is intended to represent economic conditions that set a trajectory over the course of the study. Thus the correlation of $P_{t,i}$ taken over time is strongly collinear. When considering the variance of this term, it would make little sense and obscure the underlying structure to take the variance over time. The variance over futures or games is what this term is designed to represent.

[2] Note the F, Q and L notation is simply to represent the intercept, linear and quadratic parameter positions in the equation. This notation is used throughout this appendix.

[3] The random variables in this factor do depend on time but the distribution only depends on the quarter. This is in part to represent the impacts of weather or other seasonal factors on the load. The W in the notation is simply to represent that this term is related to weather and distinguishes it from the terms in the previous equation. Later a WN notation is used to represent variables that are "weather normalized". That is functions that exclude this term.

[4] Unlike the previous term, this term does vary over time. It also varies over each future; the only control on the variance is that each quarter has a different distribution. However, there is not a built in time dependence that is the realization of $S_{t,i}$ does not depend on $S_{t-1,i}$.

$$D_{Flat}(t,i) = P_{t,i} * d_{Flat}(t)$$

Similarly the reference forecast for the weather-normalized load $d_{WN}(t)$ at time t for future i is modified as

$$D_{WN}(t,i) = P_{t,i} * S_{t,i} * d_{WN}(t)$$

2.1.2 Peak Demand Ratio

Similar to the load model a reference forecast is input into the peak demand ratio model. Let $k_r(t)$[5] be the peak ratio such that $k_r(t) * d_{Flat}(t)$ is the expected peak at time t. Then, the forecast for future i is modified by two terms like the load risk variable. The first term is

$$P_{t,i} = e^{\alpha_F \theta_{F,i} + \alpha_L \theta_{L,i}(y_t - y_0) + \alpha_Q \theta_{Q,i}(y_t - y_0)^2}$$

where $y_t = year\ at\ time\ t$; α_F, α_L and α_Q are parameters; and $\theta_{F,i} \sim \theta_{L,i} \sim \theta_{Q,i} \sim N(0,1)$, that is they are independent standard normal random variables. The second is

$$S_{t,i} = e^{\tau_{q_t} * \varepsilon_{W,i,t}}$$

where τ_{q_t} are parameters that change by quarter and $\varepsilon_{W,i,t} \sim N(0,1)$ a standard normal random variable.

Thus, the peak demand ratio for future i at time t, $K_r(t,i)$ is

$$K_r(t,i) = P_{t,i} * S_{t,i} * k_r(t)$$

2.1.3 Natural Gas Price Risk

The RPM natural gas price model modifies a reference forecast that in input into the model. Let $g(t)$ be the forecast at time t. The forecast for future i is modified by three terms. The first is

$$P_{t,i} = e^{\alpha_F \theta_{F,i} + \alpha_L \theta_{L,i}(y_t - y_0) + \alpha_Q \theta_{Q,i}(y_t - y_0)^2}$$

where $y_t = year\ at\ time\ t$; α_F, α_L and α_Q are parameters; and $\theta_{F,i} \sim \theta_{L,i} \sim \theta_{Q,i} \sim N(0,1)$, that is they are independent standard normal random variables. The second is

$$S_{t,i} = e^{\tau_{q_t} * \varepsilon_{W,i,t}}$$

where τ_{q_t} are parameters that change by quarter and $\varepsilon_{W,i,t} \sim N(0,1)$ a standard normal random variable.

The third term takes several factors that define start and end times where a "jump" factor is applied. For the factors take

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[5] The r here is to denote that this variable is a ratio and distinguish this from later use of the peak load. That is, $K_r(t,i)$ is the ratio of peak to aMW load where as $K(t,i)$ represents the peak load in MW

$$\phi_{s,1} = \beta_{i,1}$$

$$\phi_{d,1} = \eta_{i,1}$$

$$\phi_{r,1} = \eta_{i,1} e^{\omega_{i,1}}$$

$$\phi_{s,2} = \phi_{s,1} + \phi_{d,1} + \phi_{r,1} + \beta_{i,2}$$

$$\phi_{d,2} = \eta_{i,2}$$

$$\phi_{r,2} = \eta_{i,2} e^{\omega_{i,2}}$$

Then the third term is given by

$$J_{t,i} = \prod_j e^{I\{\phi_{s,j} < y_t - y_0 < \phi_{s,j} + \phi_{d,j}\}\omega_{i,j} - I\{\phi_{s,j} + \phi_{d,j} < y_t - y_0 < \phi_{s,j} + \phi_{d,j} + \phi_{r,j}\}\omega_{i,j}/\gamma_j} \qquad [6],[7]$$

Where $\beta_{i,j} \sim Unif(a_j, b_j)$, $\eta_{i,j} \sim Unif(c_j, d_j)$, and $\omega_{i,j} \sim Unif(e_j, f_j)$ and γ_j are scaling factors and a_j, b_j, c_j, d_j, e_j and f_j are parameters.

Given these three terms, the natural gas price for future i at time t is

$$G(t, i) = P_{t,i} * S_{t,i} * J_{t,i} * g(t)$$

2.1.4 Carbon Tax Risk (or Societal Damage Cost)

The RPM carbon tax risk model uses a few parameters to estimate a CO2 tax for the model.

$$C(t, i) = I_{\{t > s_i\}} * min\left(1, u_i * {}^{s_i}/_l\right) * q$$

where $s_i \sim LogN(\mu, \sigma)$ represents the first period in which the tax is applied and $u_i \sim Unif(0,1)$ and μ, σ, q and l are parameters.

2.1.5 Electricity Price Risk

The RPM electricity price model modifies a reference forecast that in input into the model. Let $m_{On}(t)$ be the on-peak electricity price forecast and $m_{Off}(t)$ be the off-peak electricity price forecast at time t. The forecast for future i is modified by two terms. The first is

$$P_{t,i} = e^{\alpha_F \theta_{F,i} + \alpha_L \theta_{L,i}(y_t - y_0) + \alpha_Q \theta_{Q,i}(y_t - y_0)^2}$$

where $y_t = year\ at\ time\ t$; α_F, α_L and α_Q are parameters; and $\theta_{F,i} \sim \theta_{L,i} \sim \theta_{Q,i} \sim N(0,1)$, that is they are independent standard normal random variables.

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[6] This factor represents a jump or temporary deviation from the baseline set by the $P_{t,i}$ factor. In market prices this in part represents speculation or other impacts that drive markets away from underlying fundamentals.
[7] Note the s, d and r notation represents terms the affect the start, duration and recovery period of a jump. These should be taken as simply representing the positions of the terms within the equation.

The second term is given by

$$S_{t,i} = e^{\tau_{q_t} * \varepsilon_{W,i,t}}$$

where τ_{q_t} are parameters that change by quarter and $\varepsilon_{W,i,t} \sim N(0,1)$ a standard normal random variable.

The third term takes several factors that define start and end times where a "jump" factor is applied. For the factors take

$$\phi_{s,1} = \beta_{i,1}$$

$$\phi_{d,1} = \eta_{i,1}$$

$$\phi_{r,1} = \eta_{i,1} e^{\omega_{i,1}}$$

$$\phi_{s,2} = \phi_{s,1} + \phi_{d,1} + \phi_{r,1} + \beta_{i,2}$$

$$\phi_{d,2} = \eta_{i,2}$$

$$\phi_{r,2} = \eta_{i,2} e^{\omega_{i,2}}$$

Then the third term is given by

$$J_{t,i} = \prod_j e^{I\{\phi_{s,j} < y_t - y_0 < \phi_{s,j} + \phi_{d,j}\} \omega_{i,j} - I\{\phi_{s,j} + \phi_{d,j} < y_t - y_0 < \phi_{s,j} + \phi_{d,j} + \phi_{r,j}\} \omega_{i,j} / \gamma_j}$$

Where $\beta_{i,j} \sim Unif(a_j, b_j)$, $\eta_{i,j} \sim Unif(c_j, d_j)$, and $\omega_{i,j} \sim Unif(e_j, f_j)$ and γ_j are scaling factors and a_j, b_j, c_j, d_j, e_j and f_j are parameters.

The fourth term scales the distribution according to the forecasts of gas price g_t, load d_t and hydro h_t. It also uses a systematic sampling of hydro $H_{t,i}$ as well as the risk model outputs for the natural gas price $G_{t,i}$ and the load $D_{t,i}$.

$$B_{t,i} = \frac{G_{t,i}^{\rho_1} e^{\rho_2 D_{t,i} + \rho_3 H_{t,i}}}{g_t^{\rho_1} e^{\rho_2 d_t + \rho_3 h_t}}$$

Where ρ_i are parameters. Given these four terms and the carbon tax price $C(t,i)$, the on-peak electricity price for the east zone for future i at time t is

$$M_{On}(t,i) = P_{t,i} * S_{t,i} * B_{t,i} * J_{t,i} * m_{On}(t) + C(t,i) * \frac{k}{2000}$$

and the off-peak electricity price for the east zone for future i at time t is

$$M_{Off}(t,i) = P_{t,i} * S_{t,i} * B_{t,i} * J_{t,i} * m_{Off}(t) + C(t,i) * \frac{k}{2000}$$

where k is a parameter representing the CO2 emissions associated with market power.

2.1.6 Renewable Energy Credit Value Risk

The RPM REC value risk model modifies a reference forecast that is input into the model. It is similar to the other risk models. The forecast, r_t, for future i is modified by two terms. The first is

$$P_{t,i} = e^{\alpha_F \varepsilon_{F,i} + \alpha_L \varepsilon_{L,i}(y_t - y_0) + \alpha_Q \varepsilon_{Q,i}(y_t - y_0)^2}$$

where $y_t = year\ at\ time\ t$; α_F, α_L and α_Q are parameters; and $\varepsilon_{F,i} \sim \varepsilon_{L,i} \sim \varepsilon_{Q,i} \sim N(0, .15)$, that is they are independent normal random variables. The second is

$$S_{t,i} = e^{\tau_{q_t} * \varepsilon_{W,i,t}}$$

where τ_{q_t} are parameters that change by quarter and $\varepsilon_{W,i,t} \sim N(0,1)$ a standard normal random variable. Given these two terms, the REC price for future i at time t, $R_{t,i}$, is

$$R_{t,i} = P_{t,i} * S_{t,i} * r_t$$

2.2 Futures Functions

The risk models produce simulation series that have time and future or game indices. In some cases these series are directly used in the simulation. However, in others the series are transformed for use in the simulation. There are also transformations of input data before it is used in the simulation. These functional transformations are documented in this section.

2.2.1 Load

With the above risk model there are four risk-informed load time series that are calculated for use in RPM. These are:

- flat electric load forecast for on-peak periods
- flat electric load forecast for off-peak periods
- weather normalized load forecast on-peak periods
- weather normalized load forecast off-peak periods.

The flat electric on-peak load at time t for future i for is

$$D_{Flat,On}(t,i) = D_{Flat}(t,i) * k_{On}(t)$$

where $k_{On}(t)$ is a forecast multiplier based on the ratio of the on-peak load to the flat load. Similarly the flat electric off-peak load at time t for future i is

$$D_{Flat,Off}(t,i) = D_{Flat}(t,i) * k_{Off}(t)$$

where $k_{Off}(t)$ is a forecast multiplier based on the ratio of the off-peak load to the flat load.

The weather-normalized on-peak load at time t for future i is

$$D_{WN,On}(t,i) = D_{WN}(t,i) * k_{On}(t)$$

and the weather-normalized off-peak load at time t for future i is

$$D_{WN,Off}(t,i) = D_{WN}(t,i) * k_{Off}(t)$$

2.2.2 Natural Gas

With the above risk model there are two risk-informed natural gas time series that are calculated for use in the RPM. These are the natural gas prices for the west zone and the natural gas prices for the east zone. The price for the west zone is simply the price calculated by the risk model that is

$$G_{West}(t,i) = G(t,i)$$

The price for the east zone is

$$G_{East}(t,i) = \max\left(v, G(t,i) - u(t)\right)$$

where v is a parameter representing the minimum price for natural gas and $u(t)$ is a forecast of the difference in price between the east and the west zones.

2.2.3 Electricity Price

With the above risk model there are four risk-informed electricity market price time series. Two are from the east zone and are given above, that is for on-peak

$$M_{East,On}(t,i) = M_{On}(t,i)$$

And for off-peak in the east zone

$$M_{East,Off}(t,i) = M_{Off}(t,i)$$

For the west zone there is an adder for both on-peak and off-peak, $W_{On}(t)$ and $W_{Off}(t)$ respectively. That is, for the west zone the on-peak electricity price is

$$M_{West,On}(t,i) = M_{On}(t,i) + W_{On}(t)$$

And for the west zone the off-peak electricity price is

$$M_{West,Off}(t,i) = M_{Off}(t,i) + W_{Off}(t)$$

2.2.4 Hydro Generation

The hydro generation time series is a function of the 80 water years. The RPM is setup to select a random water year and then proceed sequentially from that water year through the 20 year run time. That is, if $h_{West,On}(q)$ is the hydro generation for quarter q of the historic record, $1 \leq q \leq 80 * 4$ for the 80 water years. And $h_{West,Off}(q)$, $h_{East,On}(q)$ and $h_{East,Off}(q)$ are defined similarly, then the time series for on-peak hydro generation for future i at time t is

$$H_{West,On}(t,i) = h_{West,On}(j+t)$$

where $j \sim DiscreteUnif(1, 80 * 4 - 3)$. Similarly

$$H_{West,Off}(t,i) = h_{West,Off}(j+t)$$

$$H_{East,On}(t,i) = h_{East,On}(j + t)$$

$$H_{East,Off}(t,i) = h_{East,Off}(j + t)$$

3 ESTIMATING PARAMETERS FOR RPM

The RPM has many parameters and input assumptions that drive the model results.. These parameters and input assumptions are based on a variety of sources including Council forecasts, historic data and expert opinions from the Council's advisory committees and others. This section documents the process of estimating these parameters.

3.1 Load Model

The primary component used for scaling the load model is of the form

$$P_{t,i} = e^{\alpha_F \theta_{F,i} + \alpha_L \theta_{L,i}(y_t - y_0) + \alpha_Q \theta_{Q,i}(y_t - y_0)^2}$$

If you consider that the random variables $\theta_{F,i} \sim \theta_{L,i} \sim \theta_{Q,i} \sim N(0,1)$ do not depend on time then this equation can be seen as only depending on time through the year of the simulation. The load forecast for the 7th plan has a high, medium and low forecast. A regression is used to ensure that this risk model's load forecast have a range that is based on those forecasts. The parametric assumptions underlying the model require that two point estimates be used to fit the distribution. To use three parameters would require some simplification or mechanics to alter the underlying distribution that do not currently exist in the model. That is, if H_t, M_t and L_t are the high, medium and low load forecasts respective then use regression to find a, b and c in

$$\ln(H_t/M_t) = a + b(y_t - y_0) + c(y_t - y_0)^2 + \epsilon$$

Use the same procedure to assess the fit with $\ln(L_t/M_t)$. If the inverse of the fit greatly deviates from $\ln(H_t/M_t)$ then it's possible that the underlying parametric assumptions do not fit well with the forecasts.

This can be done in both cases as if the random variables are fixed values because they do not depend on time. The problem is how to alter these values to give the desired range.

While it may be possible to use a more complicated model with multiplicative errors, the easier thing is to recognize that in simple regression there is normally error around the estimation of the coefficients. If we assume that the distribution for b has zero expectation, we can take the value from the regression to be a measure of the spread. Now since $\theta_{L,i} \sim N(0,1)$

$$\alpha_L * \theta_{L,i} \sim N(0, \alpha_L)$$

We want a value where the probability of exceeding it is .85, which is the probability associated with the high load forecast. Since we have normality

$$\Pr[\alpha_L * \varepsilon_{L,i} < \alpha_L * z_{.85}] = .85$$

Thus we set

$$b = \alpha_L * z_{.85}$$

Which implies

$$\alpha_L = {}^b\!/_{z_{.85}}$$

So taking b from the regression above it is possible to construct an estimate for α_L with a specified probability of exceeding a range.

The same method applies to the values for a and c above. This allows for RPM to be directly tied to the range implied by the load forecast.

The seasonal component adds some variability based on the quarter. The factor only depends on the quarter since it is of the form

$$S_{t,i} = e^{\tau_{q_t} * \varepsilon_{W,i,t}}$$

The best way to accomplish this is to estimate seasonality based on the historic volatility. However, most the Direct Service Industries (large industrial customers served by Bonneville) no longer operate in the region. These customers operation was highly cyclical due to global commodity prices. Therefore, historical regional load data needs to be adjusted to avoid carrying forward volatility that would not occur in the future. If we assume the seasonal factor is not intended to shape, then we know the expectation for each quarter should be zero. Thus

$$\ln\!\left(S_{t,i}\right) = \tau_{q_t} * \varepsilon_{W,i,t}$$

where $\varepsilon_{W,i,t}$ is a standard normal distribution with zero expectation and τ_{q_t} is a scaling factor. So taking the adjusted history first normalize each quarter by the annual average and then normalize the resulting shapes by the average quarterly shape. That creates a sample similar to $S_{t,i}$ which can be used to estimate the standard deviation for each quarter.

3.2 Natural Gas Price Model

Similar to the load model, regression is used to estimate the annual growth component. That is, if H_t, M_t and L_t are the high, medium and low load forecasts respective then use regression to find a, b and c in

$$\ln(H_t/M_t) = a + b(y_t - y_0) + c(y_t - y_0)^2 + \epsilon$$

This is exactly the same as in the load model except that the forecasts H_t, M_t and L_t would be different and thus when you solve for a, b and c the parameters would be different.

The seasonal component adds some variability based on the quarter. The factor only depends on the quarter since it is of the form

$$S_{t,i} = e^{\tau_{q_t} * \varepsilon_{W,i,t}}$$

The best way to accomplish this is to estimate seasonality based on the historic volatility. To do this the annual average natural gas price at Henry Hub was taken starting in 1985 and each quarter's average price was used to calculate ratio of the quarterly average price to the annual average price. These factors were then collected by quarter. The standard deviation of the log of these factors is what is used to estimate the seasonal factor τ_{q_t}.

The two price models include a jump factor that simulates the risk of market-based price deviations as described above. While there are historic deviations that may fall into this type of pricing change, there are not enough to use in estimating most of the parameters for this model. Thus the inputs used are based on testing the narrative of a persistent price change that could impact decisions on constructing resources. The size of the deviation is estimated based on the largest quarterly deviation from the data used in estimating the seasonal factor.

3.3 Electricity Price Model

Similar to the load model, regression is used to estimate the annual growth component. That is, if H_t, M_t and L_t are the high, medium and low load forecasts respective then use regression to find a, b and c in

$$\ln(H_t/M_t) = a + b(y_t - y_0) + c(y_t - y_0)^2 + \epsilon$$

This is exactly the same as in the load model except that the forecasts H_t, M_t and L_t would be different and thus when you solve for a, b and c the parameters would be different.

The seasonal component adds some variability based on the quarter. The factor only depends on the quarter since it is of the form

$$S_{t,i} = e^{\tau_{q_t} * \varepsilon_{W,i,t}}$$

The best way to accomplish this is to estimate seasonality based on the historic volatility. To do this the annual average electricity price at Mid-C was taken starting in 1996 and each quarter's average price was used to calculate ratio of the quarterly average price to the annual average price. These factors were then collected by quarter. The standard deviation of the log of these factors is what is used to estimate the seasonal factor τ_{q_t}.

The two price models include a jump factor that simulates the risk of market-based price deviations as described above. While there are historic deviations that may fall into this type of pricing change, there are not enough to use in estimating most of the parameters for this model. Thus the inputs used are based on testing the narrative of a persistent price change that could impact decisions on constructing resources. The size of the deviation is estimated based on the largest quarterly deviation from the data used in estimating the seasonal factor.

The electricity price model also includes parameters that correlate the electricity price with the natural gas price, load and hydro futures. That is given the forecasts of gas g_t, load d_t and hydro h_t, the future is altered by a factor

$$B_{t,i} = \frac{G_{t,i}{}^{\rho_1} \, e^{\rho_2 D_{t,i} + \rho_3 H_{t,i}}}{g_t{}^{\rho_1} \, e^{\rho_2 d_t + \rho_3 h_t}}$$

Where hydro is $H_{t,i}$, the natural gas price is $G_{t,i}$ and the load is $D_{t,i}$. So the parameters to estimate are ρ_1, ρ_2 and ρ_3. This is done by regressing the log of the historic Mid-C on-peak price against the historic hydro, natural gas price and load.

3.4 Peak to aMW Ratio

Similar to the load model, regression is used to estimate the annual growth component. The difference with estimating the parameters for the peak is that it first must be turned into a ratio of the peak forecast associated with the high load forecast to the aMW forecast. That is in this case take $H_t = {K_{H,t}}\big/{D_{H,t}}$ where $K_{H,t}$ is the peak forecast associated with the energy (aMW) load forecast, $D_{H,t}$.

Now using regression, in the same manner as the other models, take similar definitions for M_t and L_t and find a, b and c in

$$\ln(H_t/M_t) = a + b(y_t - y_0) + c(y_t - y_0)^2 + \epsilon$$

Once again in this context the forecasts H_t, M_t and L_t would be different and thus when you solve for a, b and c the parameters would be different.

To estimate the seasonality the historic peak to aMW ratio is calculated and then the standard deviation of the log of these ratios grouped by quarter is used as the estimate for the parameter.

3.5 Table of 7th Plan Parameters

The following tables give the estimated parameters that were input into the RPM. However, these parameters are functions of the data that were used for estimates. To the extent that these data are not available in other chapters or appendices, they are available as data sets for this appendix on the Council's website for the 7th plan.

Table L - 1: Parameters from Regression Estimates

Model	Offset (a)	Linear (b)	Quadratic (c)
Load	0.01044248	0.07857033	-0.02182819
Natural Gas	0.13490437	0.01671502	-0.00016392
Electricity Price	0.08475625	0.01313602	-0.00009875
Peak to aMW Ratio	0.01246383	-0.00000600	0.00000383
REC Prices	0.30000000	0.00000000	0.00000000

Table L - 2: Parameters from Seasonality Estimates

Model	Mean	Standard Deviation (τ_{q_t})
Load	0.0000	0.0076
Natural Gas	0.0000	0.0748
Electricity Price	0.0000	0.1313
Peak to aMW Ratio	0.0000	0.0200
REC Prices	0.0000	0.1500

Table L - 3: Parameters from Jump Estimates

Model	Jump #	Start Min (a_j)	Start Max (b_j)	Duration Min (c_j)	Duration Max (d_j)	Size Min (e_j)	Size Max (f_j)	Recovery Factor (γ_j)
Natural Gas	1	0	40	0.25	8	-0.4583	0.4518	10
	2	1	41	0.25	8	-0.4583	0.4518	10
Electricity Price	1	0	40	0.25	8	-1.7186	0.7439	10
	2	4	44	0.25	8	-1.7186	0.7439	10

Table L - 4: Electricity Price Correlation Coefficients

Correlated Series	Coefficient
Gas Price (ρ_1)	1.570000
Load (ρ_2)	0.000088
Hydro (ρ_3)	-0.000049

4 RPM RESOURCE SELECTION

A significant portion of the logic for the RPM is designed to test the cost of a resource strategy. A resource strategy restricts the available resources to determine how costs would change if some resource decisions were unavailable to a planner. Still, the decision to build a resource is based on the conditions within the future being tested. For example, when a future has lower market prices, less conservation is developed relative to a future with higher market prices. This section discusses the methods for selecting which resources of those that are available in a resource strategy are acquired within each future.

4.1 Optioning Logic for Thermal and Demand Response Resources

The resource strategy determines the maximum number of a resource that can be built within a decision period. Decision periods are selected in the model by identifying particular quarters in which decisions are made to construct new thermal, renewable or demand response resources for economics or adequacy. For the 7th plan the RPM is setup so that the first 6 years have on decision period per year and the remainder of the study has a decision period every other year. When the

model forecasts that a resource will be economic then all available units are constructed. When the model forecasts an adequacy need then it constructs the number of resources required meeting adequacy standards up to the maximum number of resources. Note the model cannot construct more resources than are optioned even if this leads to not meeting adequacy standards.

4.2 Conservation Acquisition

The acquisition of conservation is based on a target price relative to a smoothed two-year average of the previous simulated market prices. The RPM does not option conservation per se, rather it buys conservation at a pace consistent with changing market conditions. However, simply purchasing conservation at market price would not test different strategies for acquisition. Thus a fixed adder to the market price is altered as part of a different resource strategy. Any available conservation that is under the smoothed market price plus the adder is purchased. This decision is made each quarter rather than each decision period.

4.2.1 Types of conservation

Two different types of conservation are modeled in the RPM. The first type is lost opportunity and the second type is discretionary (or retrofits). Lost opportunity conservation measures are measures that coincide with an event, such as constructing a new building or buying a new appliance. If this type of conservation is not acquired the next opportunity to acquire it is based on the next time that event is anticipated to occur. In the case of an event like constructing a new building that opportunity may be expected beyond the period examined in the RPM. In the case of a new appliance, it's possible that this can happen several times throughout the study.

Discretionary measures are measures that can happen at any time. Measures such as adding insulation to an existing building do not need to coincide with a particular event.

These different types of conservation take slightly different inputs. However, the general approach to modeling the available conservation in RPM for both types of conservation is similar. The inputs to the RPM are aggregated into bins where many different measures are combined. The aggregation is done by the cost of the conservation because the decision to purchase conservation is largely based on the cost. However, most of the complication in RPM is based on how much conservation is available.

4.2.2 Program Year Logic

Each bin has two hard limits imposed: a maximum available for the entire study and a maximum available by year. Beyond these limits the amount of conservation available is based on the pace at which conservation programs developed. This would likely change each future based on market conditions. Thus each bin has an associated program year for each future. That is, the program year is a function of both the bin and the future.

If the bin is purchased the first year of the future and then purchased every subsequent year of the study then the program year would be the same as the study year. However, if a bin is not purchased until the middle of the study period then it starts in the first program year. Any time it is

purchased after that, it increments to the next program year. If it becomes not cost effective at any point during a future then it stops incrementing until it becomes cost effective again.

4.3 Resources for Renewable Portfolio Standards

If sufficient resources are not optioned to meet renewable portfolio standards, then the RPM builds the cheapest resources that qualify to meet these standards. Because these requirements are known well in advance, it is not anticipated that an optioning scheme is needed. However, the RPM has an estimate of the Renewable Energy Credit (REC) banking for the resources dedicated to the RPS requirements for each state. The banking allows RECs to expire if they remain unused but it uses the oldest RECs first to meet the RPS requirements. In Oregon, where RECs do not expire, they remain in the bank indefinitely.

4.4 Adequacy Logic

Adequacy in the RPM is defined in terms of the percentage of available resource compared to load. Each existing resource block in the RPM is given a percentage of the energy that is dedicated to regional adequacy. New resources that are purchased as part of a resource strategy are considered to be 100 percent dedicated to regional adequacy. For hydro generation, values that represent critical water conditions are used. For energy the system surplus or deficit, $Surp$, is calculated as

$$Surp_e(t,i) = \sum_r NHR_{Energy,r}(t,i) + Crit_{Energy}(t,i) - D_{WN}(t,i) * (1 + ARM_e)$$

And for capacity it is

$$Surp_c(t,i) = \sum_r NHR_{Peak,r}(t,i) + Crit_{Peak}(t,i) - K(t,i) * (1 + ARM_c)$$

Where NHR represents the non-hydro resources indiced by r dedicated to the region for peak and energy, $Crit$ represents the critical water hydro generation contribution, D_{WN} represents the load, ARM represents the adequacy reserve margin for energy and capacity and K is the expected peak load.

When $Surp_e < 0$ then the optimizer is given a penalty that is added into the NPV formulation of $min(AD_e(t), |Surp_e(t,i)|) * \$6,000,000$[8] where $AD_e(t)$ is the addressable energy deficit or the energy deficit that can be covered by the available resources at time t. The capacity deficit is similarly given a penalty of $min(AD_c(t), |Surp_c(t,i)|) * \$6,000,000$.

In making a decision to build a resource the RPM has an internal forecast of the system surplus that anticipates if there will be a deficit. This forecast is projected for 17 periods and then a linear optimization is done to determine the least expensive manner to meet the adequacy standard.

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[8] The $6 Million dollar value was chosen to make sure the penalty exceeded the quarterly cost of building any available new resource. See chapter 15 for a discussion on this.

The linear optimization is setup to minimize the system cost including penalties as described above over the forecast period. In general, building resources adds cost to the NPV but reduces the penalties. The resource additions are bounded by the options in the resource strategy.

After the linear optimization is completed only the decisions from the first decision period are used to move resources from options into construction. Once an option is exercised the resource is added to the dispatch after the resource build time has passed. For example, if a resource that takes 10 quarters or 30 months to build is selected in the decision period $t = 2$ then payments for acquiring the resource are added to the NPV starting in that period but the resource would be available for dispatch in the quarter $t = 13$ and thus could reduce penalties incurred in the $t = 13\ to\ 19$ quarters of the 17 period internal forecast.

5 RPM DISPATCH METHODOLOGY

The Regional Portfolio Model uses a distributional dispatch methodology. That is, the frequency and the value of resource dispatch are based on a market price that is determined in the model..

Note: this appendix is intended to act as a compact reference for the RPM dispatch methodology. To get a much more exhaustive description of the methodology and the intent see appendices L and P in the 5th Power Plan.

5.1 Thermal Model Derivation

The premise of the dispatch in the RPM is that the dispatch of a resource is determined by how often the market price is above the variable cost of the resource. To determine this over multiple prices and costs, a distributional dispatch calculation is required.

The equations for dispatch are derived based on the value of energy. The following equation gives a mathematical expression of the value if the generator is dispatched whenever the market price exceeds the generators variable cost, in this case represented as the price of natural gas.

$$V = \sum_{h \in H} C \cdot \max(0, (p_e(h) - p_g(h)))$$

where

H is the set of hours (672 in this case)

$p_e(h)$ is the price of electricity in this hour ($/MWh)

$p_g(h)$ is the price of gas in this hour,

　　　assuming a fixed heat rate ($/MWh)

C is the capacity of the turbine (1 MW in our case)

Rearrangement gives the value in terms of the expected return from the market.

$$V = C \sum_{h \in H} \max\left(0, p_e(h) - p_g(h)\right)$$

$$= CN_H \frac{\sum_{h \in H} \max\left(0, p_e(h) - p_g(h)\right)}{N_H}$$

or

$$V = CN_H E\left[\max\left(0, p_e(h) - p_g(h)\right)\right] \qquad (3)$$

Solving the expectation takes some statistical derivation; assuming $p_e(H)$ is a random variable and X is constant, notice that

$$\int \max(0, p_e(H) - X) f\left(p_e(H)\right) dp_e(H) = \int_X^\infty (p_e(H) - X) f\left(p_e(H)\right) dp_e(H)$$

$$= \int_X^\infty p_e(H) f\left(p_e(H)\right) dp_e(H) - X \int_X^\infty f\left(p_e(H)\right) dp_e(H)$$

In the last expression, the first integral is partial expectation and the second is a survival function. Assuming $p_e(H)$ has a lognormal distribution with $E[p_e(H)] = \bar{p}_e$ and $Var[p_e(H)] = \sigma_e$ then both of these can be expressed in terms of the standard normal distribution Φ, thus

$$\int_X^\infty p_e(H) f\left(p_e(H)\right) dp_e(H) - X \int_X^\infty f\left(p_e(H)\right) dp_e(H) = \bar{p}_e \Phi\left(\frac{-\ln X + \bar{p}_e + \sigma_e^2}{\sigma_e}\right) - X\Phi\left(\frac{-\ln X + \bar{p}_e}{\sigma_e}\right)$$

A little rearrangement then gives

$$E[max(0, p_e(H) - X)] = \bar{p}_e \Phi(d) - X\Phi(d - \sigma)$$

where

$$d = \frac{-\ln X + \bar{p}_e + \sigma^2}{\sigma_e}$$

Unfortunately, this works for a constant X but it takes some more work to get to the case where you are taking the difference of distributions. So assume $p_e(H)$ is a random variable and $p_g(H)$ is a random variable, then this follows a similar derivation to Margrabe's formula. The basic idea is as follows, since these are both lognormal it follows that $\frac{p_e(H)}{p_g(H)}$ is also a lognormal and thus

$$\frac{1}{p_g(H)} \max\left(0, p_e(H) - p_g(H)\right) = \max\left(0, \frac{p_e(H)}{p_g(H)} - 1\right)$$

follows the above result. So using a similar approach with equation for value above

$$V = E\left[S_2 \max\left(0, \frac{S_1}{S_2} - 1 \right) \right]$$

$$S_2 = CN_H p_g(h)$$

$$S_1 = CN_H p_e(h)$$

The preceding equation may be evaluated explicitly and adapted for forced outages, CO2 costs and VOM:

$$V = \varepsilon = S_1 N(d_1) - S_2 N(d_2)$$

$$cf(\overline{p}_g, \overline{p}_e) = N(d_2)$$

$$d_1 = \frac{\ln(S_1 / S_2)}{\sigma} + \sigma / 2$$

$$d_2 = d_1 - \sigma$$

$$\sigma = \sqrt{\sigma_{S_1}^{\;2} + \sigma_{S_2}^{\;2} - 2\rho\sigma_{S_1}\sigma_{S_2}}$$

where

$$S_1 = CN_H(\overline{p}_e - p_{VOM})(1 - FOR)$$

$$S_2 = CN_H(\overline{p}_g + p_{CO_2})(1 - FOR)$$

p_{VOM} is the variable O & M rate (\$/MWh)

p_{CO_2} is the carbon tax penalty (\$/MWh)

σ_{S_1} is standard deviation for $\ln(S_{1,t} / S_{1,t-1}) \approx \ln(p_{e,t} / p_{e,t-1})$

σ_{S_2} is standard deviation for $\ln(S_{2,t} / S_{2,t-1}) \approx \ln(p_{g,t} / p_{g,t-1})$

ρ is the correlation in values between S_1 and S_2

FOR is the unit's forced outage rate $(0 \le FOR \le 1.0)$

5.2 Market Balancing

The previous section gives the dispatch of a thermal resource in terms of a fixed market price. However, the internal market price in the model can change and thus the dispatch would be changed. The RPM sets limits on the ability to import energy into and export energy out of the region.

If the market price from the risk model detailed above results in a dispatch that falls within this range, then there is no further alteration of the market price and the import or export of energy is determined by taking the difference between the energy produced based on the thermal dispatch and hydro dispatch and the load net of conservation and must run resources.[9]

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[99] Certain existing thermal, wind and solar resources, new solar and new wind resources are considered must-run in the RPM.

If the market price from the risk model detailed above results in a dispatch that is outside this range then the price is changed and the dispatch is recalculated. When the dispatch and the import limit falls short of the net load then the price is changed to take an average of the market price and the upper bound of the electricity price range, $325 in 2012 dollars for the 7th plan. When the dispatch minus the export limit exceeds the net load then the price is changed to take an average of the market price and the lower bound of the electricity price range, $0 in 2012 dollars for the 7th plan. If this change in the market price is not sufficient to result in a dispatch that falls within the range then the process is repeated. If after 12 iterations the dispatch still fails to fall within this range then penalties are assessed for Loss of Load or Oversupply.

6 OPTIMIZATION OF RESOURCE STRATEGIES

The first part of understanding the optimization of resource strategies is to understand the size of the sample space. There are two adders for lost opportunity and discretionary conservation. In the many scenarios evaluated, these adders were restricted to a range of $0 to $150. Since these are both continuous variables the resulting sample space is infinite because of just these values. However, continuous variables are well suited for optimization so these variables are easily handled.

Discrete variables are much more difficult to optimize because they can have non-smooth or jumpy impacts on the objective function, which in the case of RPM is the minimization of system cost. In most of the scenarios evaluated in for the Seventh Plan the RPM there are more than $1.66 * 10^{173}$ possible combinations of the discrete variables in the resource strategy. Even if you could run a billion resource strategies per second it would be impossible to explore every possible combination. With the RPM setup for the Seventh Plan, it generally takes from 20 seconds to just under a minute to run through the calculations for one resource strategy. Distributed processing is used in RPM to make these calculations in parallel. Even with distributed processing, the most combinations of potential resource strategies run for any scenario would be around 10,000. More commonly, 3,000 to 4,000 resource strategies were tested. This is an infinitesimal fraction of a percent of the possible resource strategies. To reduce the sample space investigated by the RPM, the RPM optimization is given a reasonable starting point and must look in a local region to find improvements on that starting point.

The RPM has several different algorithmic approaches available to carry out the optimization process. The most effective algorithms are what are called evolutionary algorithms. In these algorithms, a set of points in the sample space close to the starting point are taken and the results are examined at each point. The points that minimize the objective function are given more influence in selecting a new set of points in the same region. On occasion points outside the region are added in to see if the best answer lies outside the range being examined. After a certain amount of time that is either fixed by the modeler or some amount of time that shows no improvement the search is ended and the result that has the lowest value for the objective function is reported out. The advantage of this approach is that it quickly discards strategies that are extremely expensive, usually a result of heavy penalties.

Because of the complexity of the problem, it is impossible to ever verify that a result is the absolute optimal resource strategy. Thus it is important to not only use the optimization routine but to also use expert (human) judgment as to the reasonableness of the results. The model can test any change to

a resource strategy, whether made automatically through an algorithm or manually through exploration. To the extent possible both approaches were used and results were examined critically. While it is possible under any scenario that more optimal results could be yet undiscovered, it is unlikely that these results would significantly alter the understanding or narratives developed based on the close to optimal results that were used for this plan.

7 REPORTING OUTPUTS

The RPM has many variables that are calculated for each simulation. The main output is the system cost. There are many other outputs that can be reported. This section discusses some of the significant outputs used for communicating the results the scenarios run in RPM.

7.1 System Cost

The system cost is based on taking a stream of costs associated with running existing generation and the costs associated with building and running new generation. The difficultly is to appropriately value conservation under this scheme. To do this the calculation is based on the frozen efficiency load and the value of reducing the load is credited to the conservation.

The main objective function in the RPM is to minimize the system cost. However, the market price impacts many components of the portfolio cost. In general, when a resource that is constructed to serve regional load is dispatched to serve regional load, the cost to the region is based on the fixed and variable cost of that resource. When a resource is dispatched to export to the market, the system cost is reduced by the difference between the variable cost of dispatching the resource and the compensation from the market. When regional load is served by the market then the cost of purchasing power at the market price is added to the system cost.

Formulating a system cost function or objective function to generalize all these potential situations is necessary to simplify the optimization. The goal is to calculate the system cost for each time and future, that is $V_{i,t} = Net\ system\ cost\ at\ time\ t\ for\ future\ i$.

First the cost of serving the load must be considered. If the load was all served at market price then it would be:

$$\left(D_{WN,On}(t,i) * M_{West,On}(t,i) * Hrs_{On} + D_{WN,Off}(t,i) * M_{West,Off}(t,i) * Hrs_{Off}\right) * Corr$$

Where D_{WN} is the load in on and off-peak, M_{West} is the market price for the western side of the region, Hrs represents the number of hours for in the on-peak and off-peak periods and $Corr$ is an adjustment factor to represents intra-period correlation between the market price and the load.

Of course when conservation is developed, then the cost of acquisition is added to the system cost but the cost of serving load at market price or at the variable cost of a resource would be reduced. That is

$$\left(Cons_{On}(t,i) * Hrs_{On} + Cons_{Off}(t,i) * Hrs_{Off}\right) * B_{Avg}$$
$$- \left(Cons_{On}(t,i) * M_{West,On}(t,i) * Hrs_{On} + Cons_{Off}(t,i) * M_{West,Off}(t,i) * Hrs_{Off}\right)$$

Where $Cons$ describes the amount of cumulative conservation for time t and B_{Avg} is the average price per MWh for the cumulative conservation.

Both the previous terms assume only market price is used for load. To account for when load is served by dedicated generation an adjustment term needs to be included. For resources that are must run, including hydro this is done for the entire amount of generation. That is for hydro

$$Hydro(t,i) * (Hrs_{On} + Hrs_{Off}) * (M_{West,Avg}(t,i) - HydVOM)$$

Where

$$M_{West,Avg}(t,i) = \left. (M_{West,On}(t,i) * Hrs_{On} + M_{West,Off}(t,i) * Hrs_{Off}) \middle/ (Hrs_{On} + Hrs_{Off}) \right.$$

And $Hydro$ is the average hydro generation and $HydVOM$ is the variable operating and maintenance cost for hydro. This term takes the difference between the market price and the cost of running the hydro as the adjustment. Thus if a single aMW of load in the region was served by hydro dedicated to the region, the impact would be

$$(M_{West,On}(t,i) * Hrs_{On} + M_{West,Off}(t,i) * Hrs_{Off}) * Corr - (Hrs_{On} + Hrs_{Off})$$
$$* (M_{West,Avg}(t,i) - HydVOM)$$

In this case the market price "charged" to the load is "credited back" when considering the value of the hydro resource and the resulting cost is based on the $HydVOM$.

For other must run generation, the value must also account for any fuel and carbon costs. So

$$MustRun(t,i) * (Hrs_{On} + Hrs_{Off}) * (M_{West,Avg}(t,i) - MRFuel(t,i) - MRCO2(t,i) - MRVOM)$$

Where $MustRun$ is the average generation of the must run resources, $MRFuel$ represents the fuel costs for those resources, $MRCO2$ represents the carbon cost and $MRVOM$ represents the variable O&M costs.

Further extending this to dispatchable generation takes adding a term that takes the capacity factor of the generation into account. So

$$DispGen_{On}(M_{Region,On}, t, i) * Hrs_{On} * (M_{Region,On}(t,i) - DispFuel(t,i) - DispCO2(t,i) - DispVOM)$$
$$+ DispGen_{Off}(M_{Region,On}, t, i) * Hrs_{Off}$$
$$* (M_{Region,Off}(t,i) - DispFuel(t,i) - DispCO2(t,i) - DispVOM)$$

Where $DispGen_{On}$ and $DispGen_{Off}$ are the average generation dispatched by a resource during on and off peak periods, respectively. Note that the dispatch is a function of the market price. Further, $DispFuel$, $DispCO2$ and $DispVOM$ define the fuel cost, carbon cost and variable O&M costs.

This does not account for the fixed costs of adding generation to the system. The planning, construction and the fixed O&M costs must be added into the system costs. These costs differ by resource and thus are a function of the resource:

$$DispPlan(Resource, t, i) + DispConstr(Resource, t, i) + DispFOM(Resource, t, i)$$

Where $DispPlan$, $DispConstr$ and $DispFOM$ represent the planning, construction and fixed O&M costs for each $Resource$.

For reporting out of the RPM all these costs are added up to reach the net system cost, $V_{t,i}$.

Note that the optimization routine uses two other components of the system cost: the costs of not being able to balance the market and the costs of not meeting adequacy standards. These penalties help the optimization find resource strategies that avoid market imbalance and inadequate systems. However, these penalty costs are not reported in the net system cost.

7.2 Net Present Value of System Costs

Since the system costs represent a time stream of cash flows, these can be discounted to get a present value of system costs. This uses the standard formulation of net present value or NPV. That is for the net system cost $V_{t,i}$ the net present value for future i is

$$NPV_{Study,i} = \sum_{t=1}^{80} \frac{V_{t,i}}{(1 + disc)^t}$$

Where $disc$ is the discount rate for the calculation and NPV_{Study} represents the value of the study horizon, i.e. $t = 1 \ to \ 80$.

7.2.1 Perpetuity Effects

The value within the study horizon does not account for the impacts of the resource strategy that carry beyond the scope of the study. The RPM estimates these effects using calculations based on the NPV formulation. To estimate these effects there needs to be an estimate for the system cost after time $t = 80$. For the 7th plan, the last 8 periods were used to estimate the impacts to system cost going into perpetuity. That is, the estimate of for $V_{t=81,i} = V_{t=73,i}$ or more generally, $V_{t,i} = V_{t-8,i} \ for \ all \ t > 80$. Given this estimate, the NPV can be considered to infinity, that is

$$NPV_i = NPV_{Study,i} + NPV_{Perp,i} = \sum_{t=1}^{80} \frac{V_{t,i}}{(1 + disc)^t} + \sum_{t=81}^{\infty} \frac{V_{t,i}}{(1 + disc)^t}$$

Now for $t > 80$,

$$V_{t,i} = V_{t-8,i}$$

So

$$\frac{1}{(1 + disc)^t} V_{t,i} = \frac{1}{(1 + disc)^t} V_{t-8,i} = \frac{1}{(1 + disc)^8} \left[\frac{1}{(1 + disc)^{t-8}} V_{t-8,i} \right]$$

This allows the perpetuity equation to be rearranged to

$$NPV_{Perp,i} = \sum_{t=73}^{80} \left[\sum_{j=1}^{\infty} \frac{1}{(1+disc)^{8j}} \right] V_{t,i} = \left[\sum_{j=1}^{\infty} \frac{1}{(1+disc)^{8j}} \right] \sum_{t=73}^{80} V_{t,i}$$

Now it turns out that $\left[\sum_{j=1}^{\infty} \frac{1}{(1+disc)^{8j}} \right]$ is a geometric series[10] and $\frac{1}{(1+disc)^8} < 1$, so

$$\left[\sum_{j=1}^{\infty} \frac{1}{(1+disc)^{8j}} \right] \sum_{t=73}^{80} V_{t,i} = \frac{1}{1-(1+disc)^{-8}} \sum_{t=73}^{80} V_{t,i}$$

Thus

$$NPV_i = \sum_{t=1}^{80} \frac{V_{t,i}}{(1+disc)^t} + \frac{1}{1-(1+disc)^{-8}} \sum_{t=73}^{80} V_{t,i}$$

7.3 Carbon Emissions

The dispatch, detailed earlier in this chapter, determines the amount of carbon emissions from the resources in the region and resources that are contracted to serve the region that emit carbon. Also, the emissions related to imports not related to the resources in RPM are added to the regional emissions. Imports are assigned a carbon dioxide emission rate of around half a metric ton per MWh.

The RPM also has an estimate of the emissions related to the Environmental Protection Agency's Clean Power Plan (CPP). This is calculated by taking a subset of the plants in the region and estimating those emissions. Since the CPP includes only carbon dioxide emissions within state boundaries, imports are not counted in this calculation.

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[10]The convergence of a geometric series shows that $\sum_{i=0}^{\infty} x^i = 1 + x + x^2 + x^3 + \ldots = \frac{1}{(1-x)}$ for $0 < x < 1$